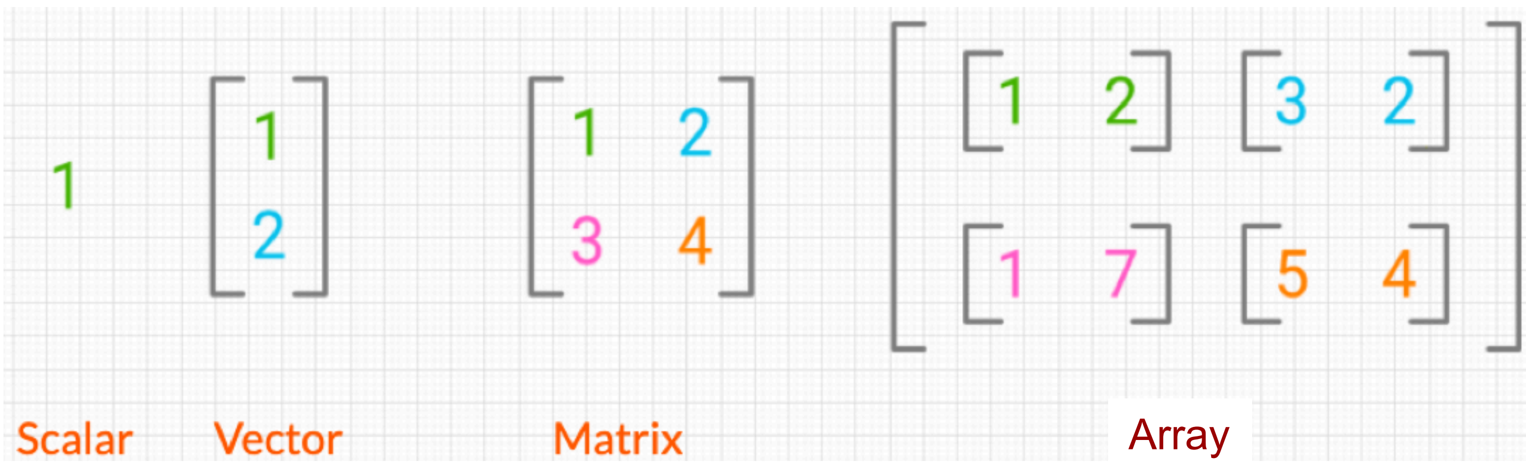
Introduction to library numpy

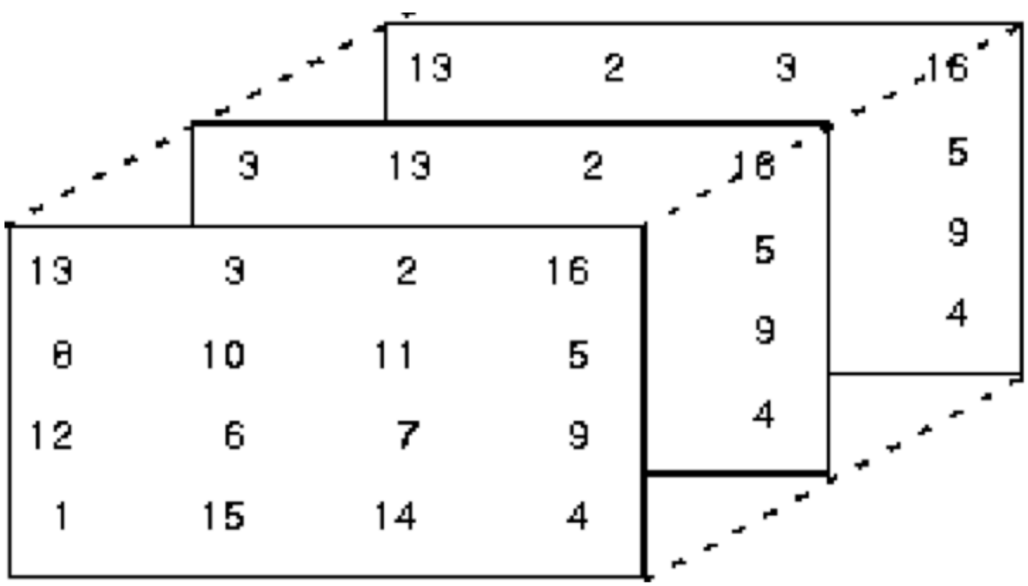
numpy

The library for working with arrays



numpy

Examples of 3D-arrays




```
array([[[5, 8, 9, 5, 0],
        [0, 1, 7, 6, 9],
        [2, 4, 5, 2, 4],
        [2, 4, 7, 7, 9]],

       [[1, 7, 0, 6, 9],
        [9, 7, 6, 9, 1],
        [0, 1, 8, 8, 3],
        [9, 8, 7, 3, 6]],

       [[5, 1, 9, 3, 4],
        [8, 1, 4, 0, 3],
        [9, 2, 0, 4, 9],
        [2, 7, 7, 9, 8]]])
```

numpy

Used to perform math operations (product, inverse, transpose), transformations, on arrays

numpy array

The main data structure is the array (ndarray)

It is a grid of numeric values of the same data type

- vectors — 1D arrays
- matrices — 2D arrays
- high-dimensional arrays — nD arrays

numpy array

We will review

- how to create arrays
- array attributes
- how to transform arrays
- matrix operations

shape of 1D array

```
import numpy as np
```

```
x = np.array([5, 1, 2, 4, 5, 1, 5])
x
```

```
array([5, 1, 2, 4, 5, 1, 5])
```

convert a list to a 1D array

```
x.shape
```

```
(7,)
```

Creating a 2D array

```
import numpy as np
```

```
x = np.array([[1, 3, 3], [1, 4, 3], [1, 3, 4]])
x
```

```
array([[1, 3, 3],
       [1, 4, 3],
       [1, 3, 4]])
```

shape of a 2D array

```
import numpy as np
```

```
x = np.array([[1, 3, 3], [1, 4, 3], [1, 3, 4]])
x
```

```
array([[1, 3, 3],
       [1, 4, 3],
       [1, 3, 4]])
```

```
x.shape
```

```
(3, 3)
```

np functions to create ndarrays

Function	Description
np.array	Convert input data (list, tuple, array, or other sequence type) to an ndarray either by inferring a dtype or explicitly specifying a dtype; copies the input data by default
np.arange	Like the built-in `range` but returns an ndarray
np.ones	Produce an array of all 1s
np.zeros	Produce an array of all 0s
np.empty	Create new arrays by allocating new memory, but do not populate with any values
np.full	Produce an array of the given shape and dtype with all values set to the indicated "fill value"
np.eye, identity	Create a square N × N identity matrix (1s on the diagonal and 0s elsewhere)

INTRODUCTION – Creating ndarrays

```
# create a list
```

```
x = [5,1,2,4,5,1,5]
x
```

```
[5, 1, 2, 4, 5, 1, 5]
```

```
len(x)
```

```
7
```

```
# convert it to an 1D-array
```

```
x = np.array(x)
x
```

```
array([5, 1, 2, 4, 5, 1, 5])
```

```
x.shape
```

```
(7,)
```

INTRODUCTION – Creating ndarrays

```python
# create a list
```

```python
x = [5,1,2,4,5,1,5]
x
```

```
[5, 1, 2, 4, 5, 1, 5]
```

```python
len(x)
```

```
7
```

```python
# convert it to an 1D-array
```

```python
x = np.array(x)
x
```

```
array([5, 1, 2, 4, 5, 1, 5])
```

```python
x.shape
```

```
(7,)
```

```python
# nested list (list of lists)
```

```python
x = [[1,3,3],[1,4,3],[1,3,4]]
x
```

```
[[1, 3, 3], [1, 4, 3], [1, 3, 4]]
```

```python
len(x)
```

```
3
```

```python
# convert it to an nD-array
```

```python
x = np.array(x)
x
```

```
array([[1, 3, 3],
       [1, 4, 3],
       [1, 3, 4]])
```

```python
x.shape
```

```
(3, 3)
```

INTRODUCTION – Creating ndarrays

```
x = list(range(1,10))
x
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
x = np.array(x)
x
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

INTRODUCTION – Creating ndarrays

```
x = list(range(1,10))
x
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
x = np.array(x)
x
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
x = np.arange(1,10)
x
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

INTRODUCTION – Creating ndarrays

```
x = list(range(1,10))
x
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
x = np.array(x)
x
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
x = np.arange(1,10)
x
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
x = np.arange(0,10,step=2)
x
```

```
array([0, 2, 4, 6, 8])
```

```
np.linspace(0,10,5)        # five values evenly spaced in (0,10)
```

```
array([ 0. ,  2.5,  5. ,  7.5, 10. ])
```

INTRODUCTION – Commonly used arrays

```
np.zeros(10)
```

```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

```
np.zeros(10,dtype=int)
```

```
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

```
np.eye(3)                    # 3x3 identity matrix
```

```
array([[1., 0., 0.],
       [0., 1., 0.],
       [0., 0., 1.]])
```

INTRODUCTION – Commonly used arrays

```
np.eye(3)                       # 3x3 identity matrix
```

```
array([[1., 0., 0.],
       [0., 1., 0.],
       [0., 0., 1.]])
```

```
x = np.empty((3,3))             # Think of a matrix with NAs
x
```

```
array([[1., 0., 0.],
       [0., 1., 0.],
       [0., 0., 1.]])
```

INTRODUCTION – Commonly used arrays

```
np.ones((3,4),dtype=float)    # 3x4 array of ones
```

```
array([[1., 1., 1., 1.],
       [1., 1., 1., 1.],
       [1., 1., 1., 1.]])
```

```
np.full((3,4),2.1)    # 3x4 array filled with same number
```

```
array([[2.1, 2.1, 2.1, 2.1],
       [2.1, 2.1, 2.1, 2.1],
       [2.1, 2.1, 2.1, 2.1]])
```

INTRODUCTION – Creating arrays with random values

Values from the uniform distribution

- UNIF(0,1) np.random.random(shape)
- UNIF(a,b) np.random.uniform(a,b,shape)
- DUNIF(a,b) np.random.randint(a,b,shape)

Values from the normal distribution

- N(0,1) np.random.randn(shape)
- N(μ,σ) np.random.normal(μ,σ,shape)

INTRODUCTION – Creating arrays with Uniform values

```
np.random.seed(9)
```

```
# U(0,1) and U(a,b)
```

```
x = np.random.random(size = (2,3))
x
```

fill the 2x3 array with U(0,1) values

```
array([[0.01037415, 0.50187459, 0.49577329],
       [0.13382953, 0.14211109, 0.21855868]])
```

```
x = np.random.uniform(low=100,high=200,size = (2,3))
x
```

fill the 2x3 array with U(100,200) values

```
array([[141.85081805, 124.81011684, 108.40596512],
       [134.54986401, 116.67763465, 187.85590855]])
```

INTRODUCTION – Array with Discrete Uniform values

```
# DUNIF(5,10,(3,4))
```

fill with integers from 5 to 9

```
np.random.randint(low=5,high=10,size=(3,4))
```

```
array([[8, 5, 7, 5],
       [7, 9, 5, 9],
       [7, 5, 6, 5]])
```

3x3 Array with Standard Normal values

```
# N(0,1)
```

```
x = np.random.randn(3,3)
x
```

```
array([[ 0.63589108,  1.74011731,  0.29668222],
       [ 0.70750366,  1.82281576,  0.43076903],
       [ 1.54272963, -0.90072117, -0.13712501]])
```

3x2 Array with Non-standard Normal values

```
# N(mean=10,s.deviation = 3)
```

```
np.random.normal(loc=10,scale=3,size=(3,2))
```

```
array([[13.89273704, 12.0258135 ],
       [10.09587435, 12.75443769],
       [11.1415284 , 11.54910246]])
```

array attributes

INTRODUCTION – array attributes

```
np.random.seed(1)
x = np.random.randint(10,size=(3,4,5))
x
```

```
array([[[5, 8, 9, 5, 0],
        [0, 1, 7, 6, 9],
        [2, 4, 5, 2, 4],
        [2, 4, 7, 7, 9]],

       [[1, 7, 0, 6, 9],
        [9, 7, 6, 9, 1],
        [0, 1, 8, 8, 3],
        [9, 8, 7, 3, 6]],

       [[5, 1, 9, 3, 4],
        [8, 1, 4, 0, 3],
        [9, 2, 0, 4, 9],
        [2, 7, 7, 9, 8]]])
```

```
np.random.randint(low=5,high=10,size=(3,4))
```

```
array([[8, 5, 7, 5],
       [7, 9, 5, 9],
       [7, 5, 6, 5]])
```

INTRODUCTION – ndarrays attributes

```
np.random.seed(1)
x = np.random.randint(10,size=(3,4,5))
x
```

```
array([[[5, 8, 9, 5, 0],
        [0, 1, 7, 6, 9],
        [2, 4, 5, 2, 4],
        [2, 4, 7, 7, 9]],

       [[1, 7, 0, 6, 9],
        [9, 7, 6, 9, 1],
        [0, 1, 8, 8, 3],
        [9, 8, 7, 3, 6]],

       [[5, 1, 9, 3, 4],
        [8, 1, 4, 0, 3],
        [9, 2, 0, 4, 9],
        [2, 7, 7, 9, 8]]])
```

```
x.ndim
```

```
3
```

```
x.shape
```

```
(3, 4, 5)
```

```
x.size
```

```
60
```

INTRODUCTION – ndarrays attributes

```
np.random.seed(1)
x = np.random.randint(10,size=(3,4,5))
x
```

```
array([[[5, 8, 9, 5, 0],
        [0, 1, 7, 6, 9],
        [2, 4, 5, 2, 4],
        [2, 4, 7, 7, 9]],

       [[1, 7, 0, 6, 9],
        [9, 7, 6, 9, 1],
        [0, 1, 8, 8, 3],
        [9, 8, 7, 3, 6]],

       [[5, 1, 9, 3, 4],
        [8, 1, 4, 0, 3],
        [9, 2, 0, 4, 9],
        [2, 7, 7, 9, 8]]])
```

```
x.ndim
```

```
3
```

```
x.shape
```

```
(3, 4, 5)
```

```
x.size
```

```
60
```

```
x[0,0,2]
```

```
9
```

matrix 0, row 0, col 2

INTRODUCTION – Changing the shape of arrays

1D array

```
x = list(range(1,10))
x
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
array1D = np.array(x)
array1D
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
array1D.shape
```

```
(9,)
```

```
array1D.ndim
```

```
1
```

INTRODUCTION – reshaping arrays

1D array

```
x = list(range(1,10))
x
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
array1D = np.array(x)
array1D
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
array1D.shape
```

```
(9,)
```

```
array1D.ndim
```

```
1
```

Convert this 1D array to 2D array

```
# reshape with -1 to keep dimension (9,)
```

```
array2D = array1D.reshape((-1,1))
array2D
```

```
array([[1],
       [2],
       [3],
       [4],
       [5],
       [6],
       [7],
       [8],
       [9]])
```

```
array2D.shape
```

```
(9, 1)
```

```
array2D.ndim
```

```
2
```

INTRODUCTION – reshaping arrays

1D array

```
x = list(range(1,10))
x
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
array1D = np.array(x)
array1D
```

```
array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
array1D.shape
```

```
(9,)
```

different shapes

```
array1D.ndim
```

```
1
```

Convert this 1D array to 2D array

```
# reshape with -1 to keep dimension (9,)
```

```
array2D = array1D.reshape((-1,1))
array2D
```

```
array([[1],
       [2],
       [3],
       [4],
       [5],
       [6],
       [7],
       [8],
       [9]])
```

```
array2D.shape
```

```
(9, 1)
```

```
array2D.ndim
```

```
2
```

INTRODUCTION – concatenate arrays

```
array2
```

```
array([[1, 2, 3, 4],
       [5, 6, 7, 8]])
```

```
# concatenate
```

```
np.concatenate([array2,array2],axis = 0)
```

```
array([[1, 2, 3, 4],
       [5, 6, 7, 8],
       [1, 2, 3, 4],
       [5, 6, 7, 8]])
```

INTRODUCTION – concatenate arrays

```
array2
```

```
array([[1, 2, 3, 4],
       [5, 6, 7, 8]])
```

```
# concatenate
```

```
np.concatenate([array2,array2],axis = 0)
```

```
array([[1, 2, 3, 4],
       [5, 6, 7, 8],
       [1, 2, 3, 4],
       [5, 6, 7, 8]])
```

```
np.concatenate([array2,array2],axis = 1)
```

```
array([[1, 2, 3, 4, 1, 2, 3, 4],
       [5, 6, 7, 8, 5, 6, 7, 8]])
```

INTRODUCTION – concatenate arrays

```
array2
```

```
array([[1, 2, 3, 4],
       [5, 6, 7, 8]])
```

```
# concatenate
```

```
np.concatenate([array2,array2],axis = 0)
```

```
array([[1, 2, 3, 4],
       [5, 6, 7, 8],
       [1, 2, 3, 4],
       [5, 6, 7, 8]])
```

```
np.concatenate([array2,array2],axis = 1)
```

```
array([[1, 2, 3, 4, 1, 2, 3, 4],
       [5, 6, 7, 8, 5, 6, 7, 8]])
```

matrix operations

INTRODUCTION – matrix operations

```
c = np.array([[4, 3], [2, 1]])
c
```

```
array([[4, 3],
       [2, 1]])
```

```
d = np.array([[1, 2], [3, 4]])
d
```

```
array([[1, 2],
       [3, 4]])
```

INTRODUCTION – matrix operations

```
c = np.array([[4, 3], [2, 1]])
c
```

```
array([[4, 3],
       [2, 1]])
```

```
d = np.array([[1, 2], [3, 4]])
d
```

```
array([[1, 2],
       [3, 4]])
```

```
c*d
```

```
array([[4, 6],
       [6, 4]])
```

INTRODUCTION – matrix operations

Elementwise multiplication

```
c = np.array([[4, 3], [2, 1]])
c
```

```
array([[4, 3],
       [2, 1]])
```

```
d = np.array([[1, 2], [3, 4]])
d
```

```
array([[1, 2],
       [3, 4]])
```

```
c*d
```

```
array([[4, 6],
       [6, 4]])
```

INTRODUCTION – matrix operations

Elementwise multiplication

```
c = np.array([[4, 3], [2, 1]])
c
```

```
array([[4, 3],
       [2, 1]])
```

```
d = np.array([[1, 2], [3, 4]])
d
```

```
array([[1, 2],
       [3, 4]])
```

```
c*d
```

```
array([[4, 6],
       [6, 4]])
```

INTRODUCTION – matrix operations

Elementwise multiplication

```
c = np.array([[4, 3], [2, 1]])
c
```

```
array([[4, 3],
       [2, 1]])
```

```
d = np.array([[1, 2], [3, 4]])
d
```

```
array([[1, 2],
       [3, 4]])
```

```
c*d
```

```
array([[4, 6],
       [6, 4]])
```

INTRODUCTION – matrix operations

Elementwise multiplication

```
c = np.array([[4, 3], [2, 1]])
c
```

```
array([[4, 3],
       [2, 1]])
```

```
d = np.array([[1, 2], [3, 4]])
d
```

```
array([[1, 2],
       [3, 4]])
```

```
c*d
```

```
array([[4, 6],
       [6, 4]])
```

INTRODUCTION – Create two arrays, a and b

```
x = np.arange(1,7)
x
```

```
array([1, 2, 3, 4, 5, 6])
```

```
a = x.reshape(2,3)
a
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
b = x.reshape(3,2)
b
```

```
array([[1, 2],
       [3, 4],
       [5, 6]])
```

INTRODUCTION – matrix multiplication

```
x = np.arange(1,7)
x
```

```
array([1, 2, 3, 4, 5, 6])
```

```
a = x.reshape(2,3)
a
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
b = x.reshape(3,2)
b
```

```
array([[1, 2],
       [3, 4],
       [5, 6]])
```

```
a.dot(b)
```

```
array([[22, 28],
       [49, 64]])
```

INTRODUCTION – matrix multiplication

```
x = np.arange(1,7)
x
```

```
array([1, 2, 3, 4, 5, 6])
```

```
a = x.reshape(2,3)
a
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
b = x.reshape(3,2)
b
```

```
array([[1, 2],
       [3, 4],
       [5, 6]])
```

```
a.dot(b)
```

```
array([[22, 28],
       [49, 64]])
```

1(1) + 2(3) + 3(5) = 22

INTRODUCTION – matrix multiplication

```
x = np.arange(1,7)
x
```

```
array([1, 2, 3, 4, 5, 6])
```

```
a = x.reshape(2,3)
a
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
b = x.reshape(3,2)
b
```

```
array([[1, 2],
       [3, 4],
       [5, 6]])
```

```
a.dot(b)
```

```
array([[22, 28],
       [49, 64]])
```

1(2) + 2(4) + 3(6) = 28

INTRODUCTION – matrix multiplication

```
x = np.arange(1,7)
x
```

```
array([1, 2, 3, 4, 5, 6])
```

```
a = x.reshape(2,3)
a
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
b = x.reshape(3,2)
b
```

```
array([[1, 2],
       [3, 4],
       [5, 6]])
```

4(1) + 5(3) + 6(5) = 49

```
a.dot(b)
```

```
array([[22, 28],
       [49, 64]])
```

INTRODUCTION – matrix multiplication notation

```
x = np.arange(1,7)
x
```

```
array([1, 2, 3, 4, 5, 6])
```

```
a = x.reshape(2,3)
a
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
b = x.reshape(3,2)
b
```

```
array([[1, 2],
       [3, 4],
       [5, 6]])
```

```
a.dot(b)
```

```
array([[22, 28],
       [49, 64]])
```

```
np.dot(a,b)
```

```
array([[22, 28],
       [49, 64]])
```

INTRODUCTION – Transpose of a matrix

```
x = np.arange(1,7)
x
```

```
array([1, 2, 3, 4, 5, 6])
```

```
a = x.reshape(2,3)
a
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
a.T
```

Transpose of matrix a

```
array([[1, 4],
       [2, 5],
       [3, 6]])
```

INTRODUCTION

1D array

```
# x'x is the sum of squares of x

x

array([1, 2, 3, 4, 5, 6])

np.dot(x.T, x)

91
```

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$

INTRODUCTION – Create a square matrix from a vector

```
x = np.arange(1,10)
x[2] = 0
x
```

```
array([1, 2, 0, 4, 5, 6, 7, 8, 9])
```

```
x = x.reshape(3,3)
x
```

```
array([[1, 2, 0],
       [4, 5, 6],
       [7, 8, 9]])
```

INTRODUCTION – operations on square matrices

```python
x = np.arange(1,10)
x[2] = 0
x
```

```
array([1, 2, 0, 4, 5, 6, 7, 8, 9])
```

```python
x = x.reshape(3,3)
x
```

```
array([[1, 2, 0],
       [4, 5, 6],
       [7, 8, 9]])
```

```python
np.diag(x)
```

```
array([1, 5, 9])
```

```python
np.trace(x)
```

```
15
```

```python
np.linalg.det(x)
```

```
9.000000000000002
```

```python
from numpy import linalg
```

```python
linalg.det(x)
```

```
9.000000000000002
```

INTRODUCTION – operations on square matrices

```
x = np.arange(1,10)
x[2] = 0
x
```

```
array([1, 2, 0, 4, 5, 6, 7, 8, 9])
```

```
x = x.reshape(3,3)
x
```

```
array([[1, 2, 0],
       [4, 5, 6],
       [7, 8, 9]])
```

```
y = linalg.inv(x)
y
```

inverse of matrix x

```
array([[-0.33333333, -2.        ,  1.33333333],
       [ 0.66666667,  1.        , -0.66666667],
       [-0.33333333,  0.66666667, -0.33333333]])
```

```
np.diag(x)
```

```
array([1, 5, 9])
```

```
np.trace(x)
```

```
15
```

```
np.linalg.det(x)
```

```
9.000000000000002
```

```
from numpy import linalg
```

```
linalg.det(x)
```

```
9.000000000000002
```

Example

Linear Regression

Linear Regression

Use numpy, sklearn, and matplotlib to

- Display the relation between two variables x, y
- Find the regression line
- Predict y given X
- Draw a scatterplot with the regression line

`

Linear Regression

- Car dealers use the *Red Book* to estimate the price of used cars
- This book (published monthly) lists the trade-in prices for all basic models of cars
- However, the Red Book does not indicate the value determined by the odometer reading, despite the fact that a critical factor for used-car buyers is how far the car has been driven

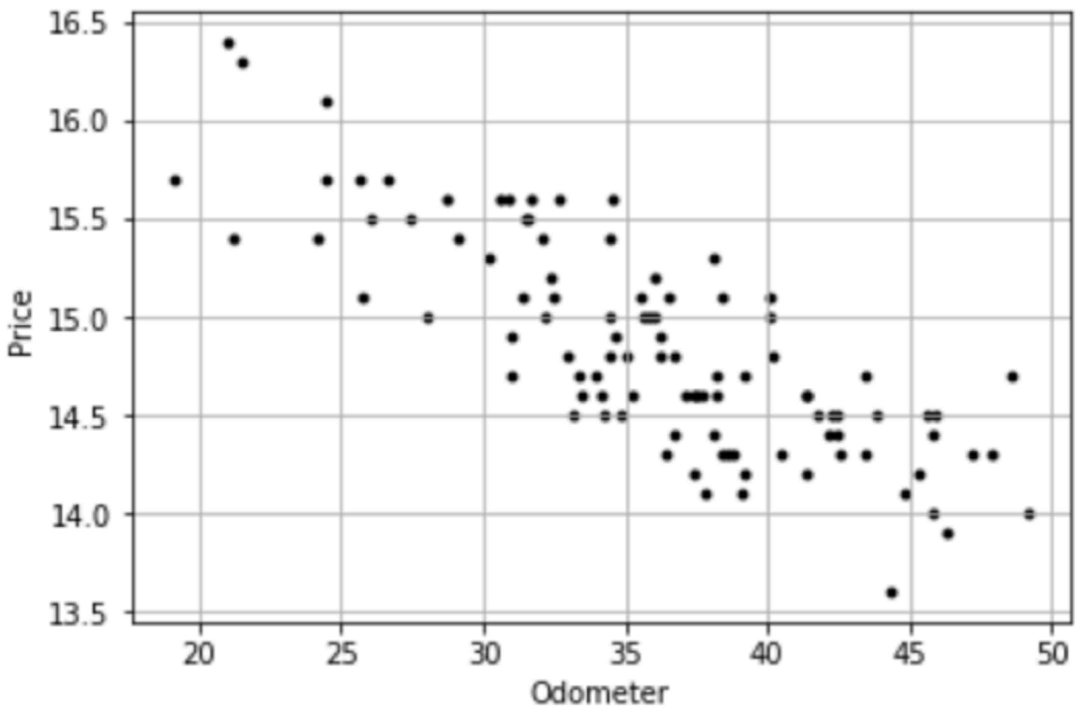
`

Linear Regression

- To examine this issue, a used-car dealer randomly selected 100 three-year old Toyota Camrys that were sold at an auction during the past month.
- The dealer recorded the price (000s) and the number of miles (000s) on the odometer (Odometer.csv)
- How much does the number of miles affect the price of a used-car?

Odometer.csv

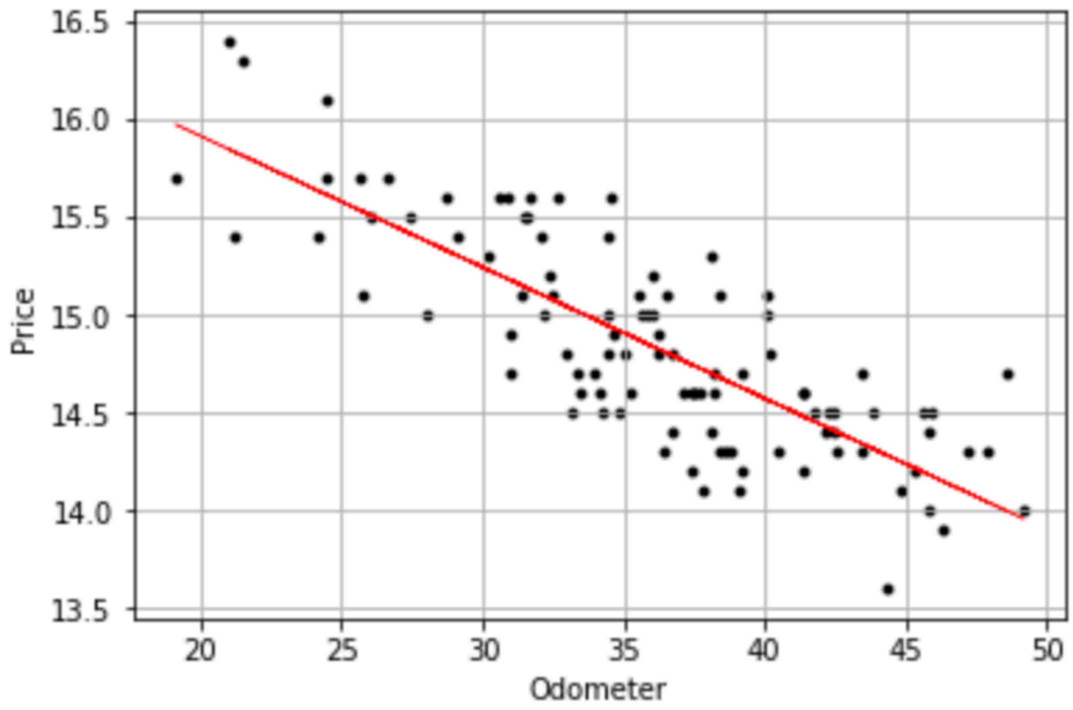
	A	B
1	Odometer	Price
2	37.4	14.6
3	44.8	14.1
4	45.8	14
5	30.9	15.6
6	31.7	15.6
7	34	14.7
8	45.9	14.5
9	19.1	15.7
10	40.1	15.1
11	40.2	14.8



100 x 2

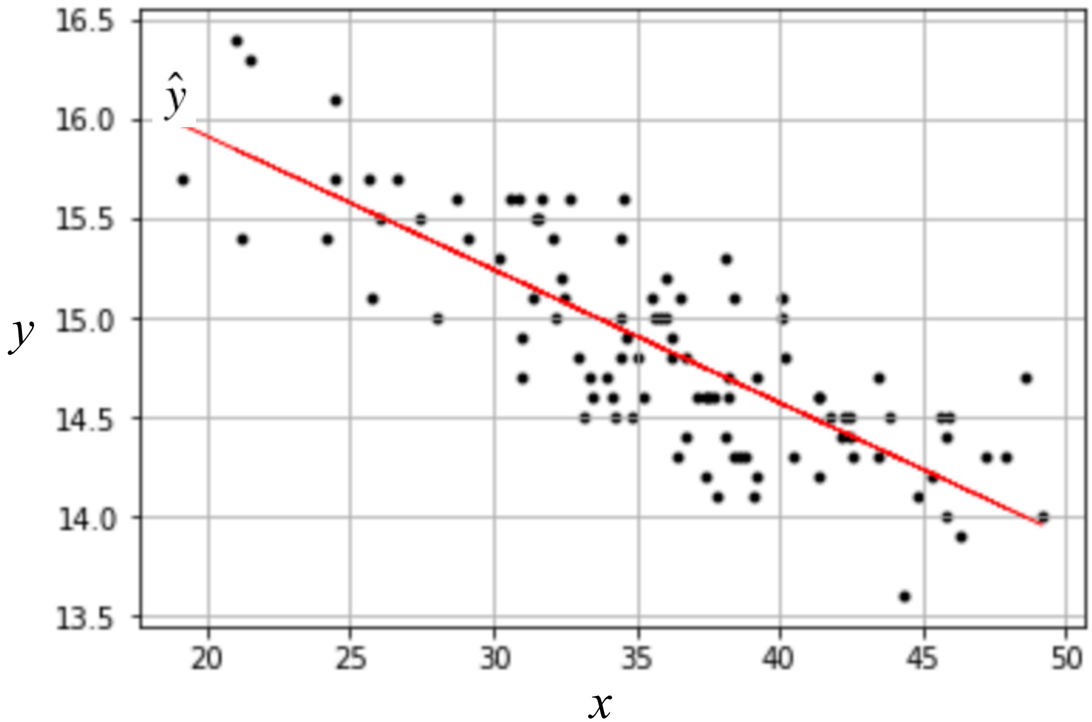
Odometer.csv

	A	B
1	Odometer	Price
2	37.4	14.6
3	44.8	14.1
4	45.8	14
5	30.9	15.6
6	31.7	15.6
7	34	14.7
8	45.9	14.5
9	19.1	15.7
10	40.1	15.1
11	40.2	14.8



100 x 2

Linear Regression



intercept slope

$$\hat{y} = b_0 + b_1 x = 17.250 - 0.0669x$$

$$b = [\ b_0\ ,\ b_1\ ]$$

$$= [\ 17.25,\ -0.0669]$$

Odometer.csv

	A	B
1	Odometer	Price
2	37.4	14.6
3	44.8	14.1
4	45.8	14
5	30.9	15.6
6	31.7	15.6
7	34	14.7
8	45.9	14.5
9	19.1	15.7
10	40.1	15.1
11	40.2	14.8

X y

headers

Reading csv file into an array

```
import numpy as np
import matplotlib.pyplot as plt
```

```
array1 = np.genfromtxt('Odometer.csv',skip_header=1, delimiter=',')
array1[:5]
```

```
array([[37.4, 14.6],
       [44.8, 14.1],
       [45.8, 14. ],
       [30.9, 15.6],
       [31.7, 15.6]])
```

```
array1.shape
```

```
(100, 2)
```

	A	B
1	Odometer	Price
2	37.4	14.6
3	44.8	14.1
4	45.8	14
5	30.9	15.6
6	31.7	15.6

X y

Reading csv file with numpy

```
import numpy as np
import matplotlib.pyplot as plt
```

```
array1 = np.genfromtxt('Odometer.csv',skip_header=1, delimiter=',')
array1[:5]
```

```
         X      y
array([[37.4, 14.6],
       [44.8, 14.1],
       [45.8, 14. ],
       [30.9, 15.6],
       [31.7, 15.6]])
```

```
array1.shape
```

```
(100, 2)
```

Reading csv file with numpy

```
import numpy as np
import matplotlib.pyplot as plt
```

```
array1 = np.genfromtxt('Odometer.csv',skip_header=1, delimiter=',')
array1[:5]
```

```
        x       y
array([[37.4, 14.6],
       [44.8, 14.1],
       [45.8, 14. ],
       [30.9, 15.6],
       [31.7, 15.6]])
```

```
array1.shape
```

```
(100, 2)
```

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

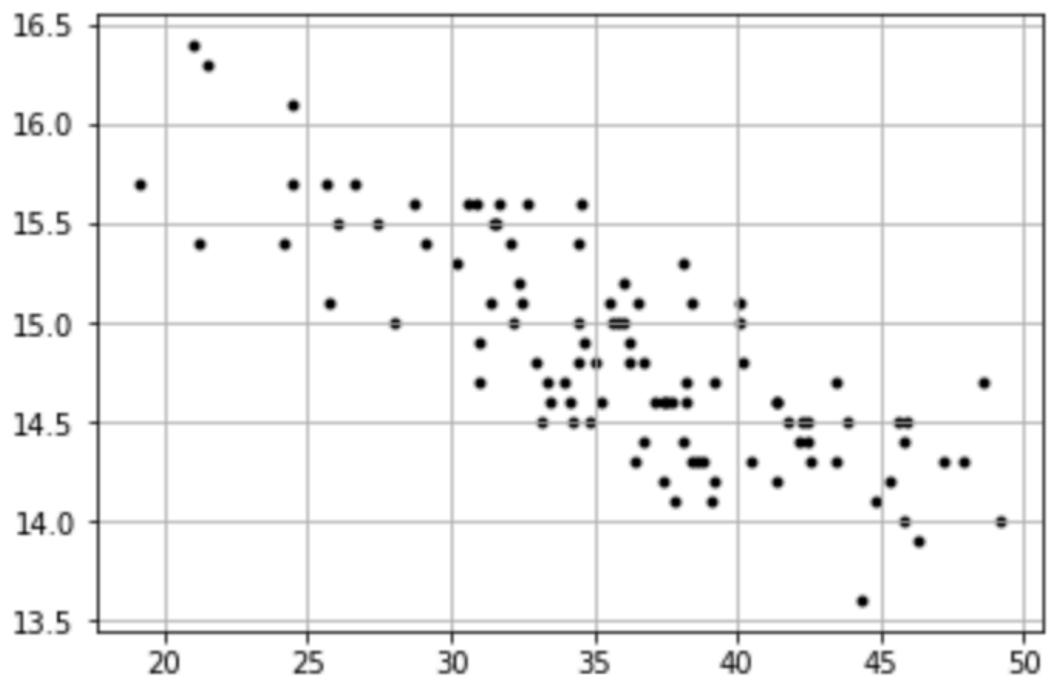
```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

1D arrays

Scatterplot with matplotlib

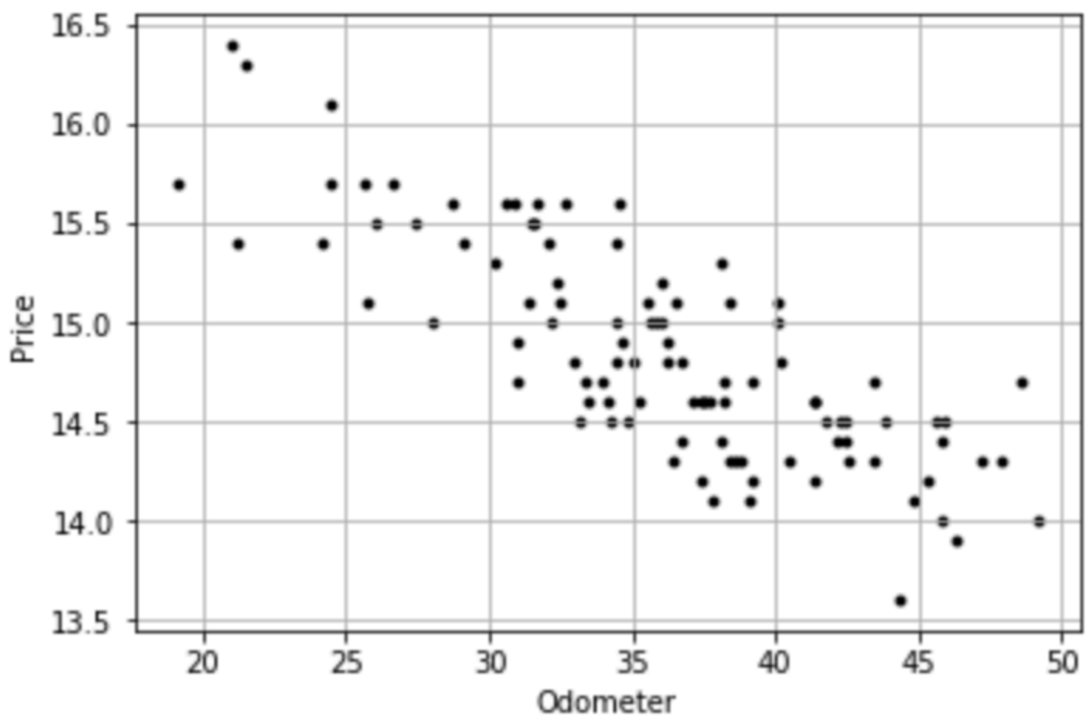
```
plt.figure()
plt.scatter(x,y,c='k',s=9)
```

```
plt.grid()
```



Scatterplot with matplotlib

```
plt.figure()
plt.scatter(x,y,c='k',s=9)
plt.ylabel('Price')
plt.xlabel('Odometer')
plt.grid()
```



Linear Regression
with library sklearn

Linear regression with sklearn

```
from sklearn.linear_model import LinearRegression
```

```
x.shape
```

```
(100,)
```

for sklearn
we need x
to be 2D array

Linear regression with sklearn

```
from sklearn.linear_model import LinearRegression
```

```
x.shape
```

```
(100,)
```

1D array

```
x_2d = x.reshape(-1, 1)
x_2d.shape
```

```
(100, 1)
```

2D array

```
m1 = LinearRegression().fit(x_2d,y)
```

Linear regression with sklearn

```
from sklearn.linear_model import LinearRegression
```

```
x.shape
```

```
(100,)
```

1D array

```
x_2d = x.reshape(-1, 1)
x_2d.shape
```

```
(100, 1)
```

2D array

```
m1 = LinearRegression().fit(x_2d,y)
```

for multiple regression with *p* predictors, this array should be of (100, *p*) shape

Linear regression with sklearn

```python
from sklearn.linear_model import LinearRegression
```

```python
x.shape
```

```
(100,)
```

```python
x_2d = x.reshape(-1, 1)
x_2d.shape
```

```
(100, 1)
```

```python
m1 = LinearRegression().fit(x_2d,y)
```

```python
m1.intercept_
```

```
17.24872734291551
```

```python
m1.coef_
```

```
array([-0.06686089])
```

$$b = [\, b_0 \,,\, b_1 \,]$$

$$= [\, 17.25,\ -0.0669]$$

Linear regression with sklearn

Predict the price of a car with Odometer reading 40

```
newval = np.array([[40]])
newval
```

```
array([[40]])
```

```
newval.shape
```

```
(1, 1)
```

newval must be a 2D numpy array

`

Predicting with model m1

Predict the price of a car with Odometer reading 40

```
newval = np.array([[40]])
newval
```

```
array([[40]])
```

```
newval.shape
```

```
(1, 1)
```

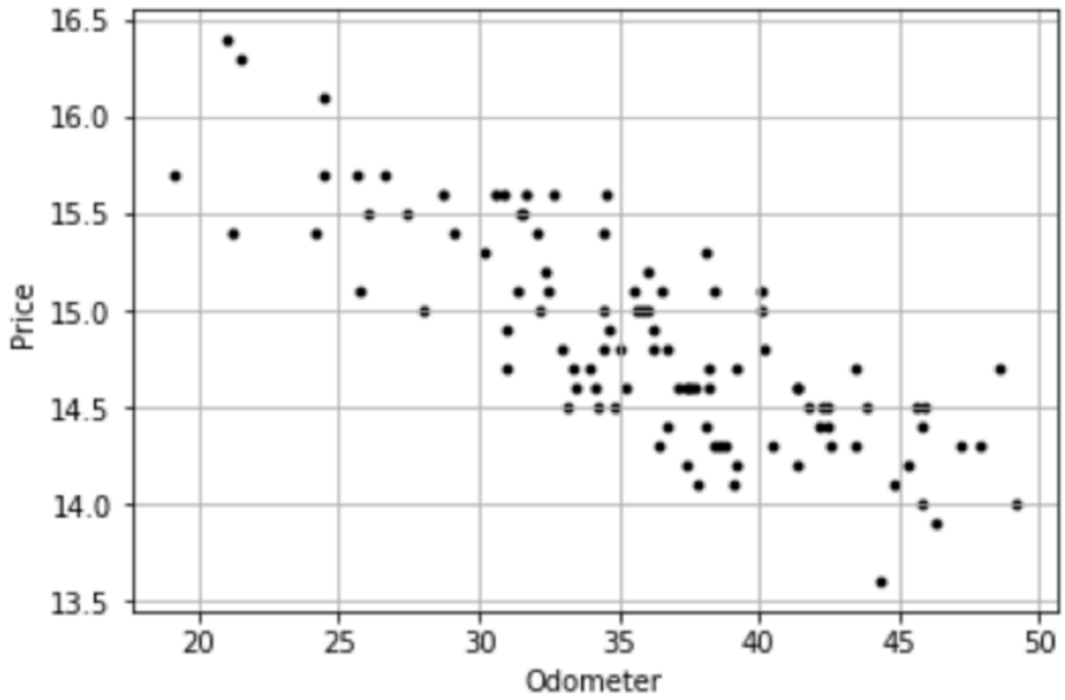
```
m1.predict(newval)
```

```
array([14.57429193])
```

car price is predicted to be 14,574 dollars

Scatterplot

```
plt.figure()
plt.scatter(x,y,c='k',s=9)
```

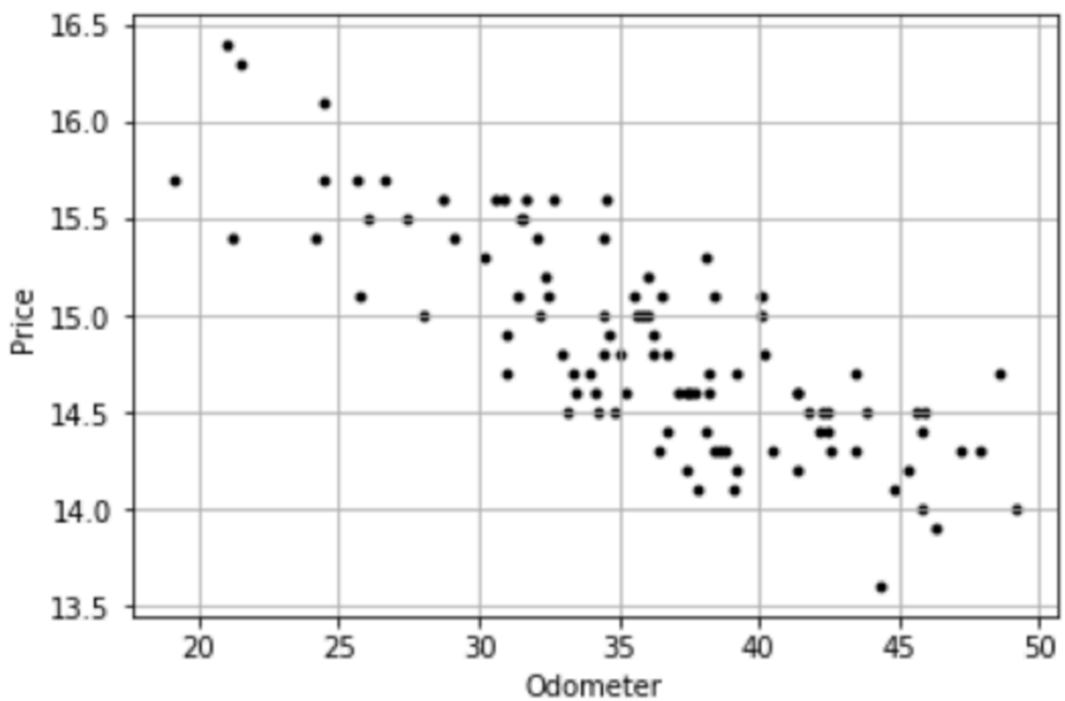


Scatterplot with regression line

```
# predict price
# for all Odometer values in x
```

```
yhat = m1.predict(x_2d)
```

```
plt.figure()
plt.scatter(x,y,c='k',s=9)
```

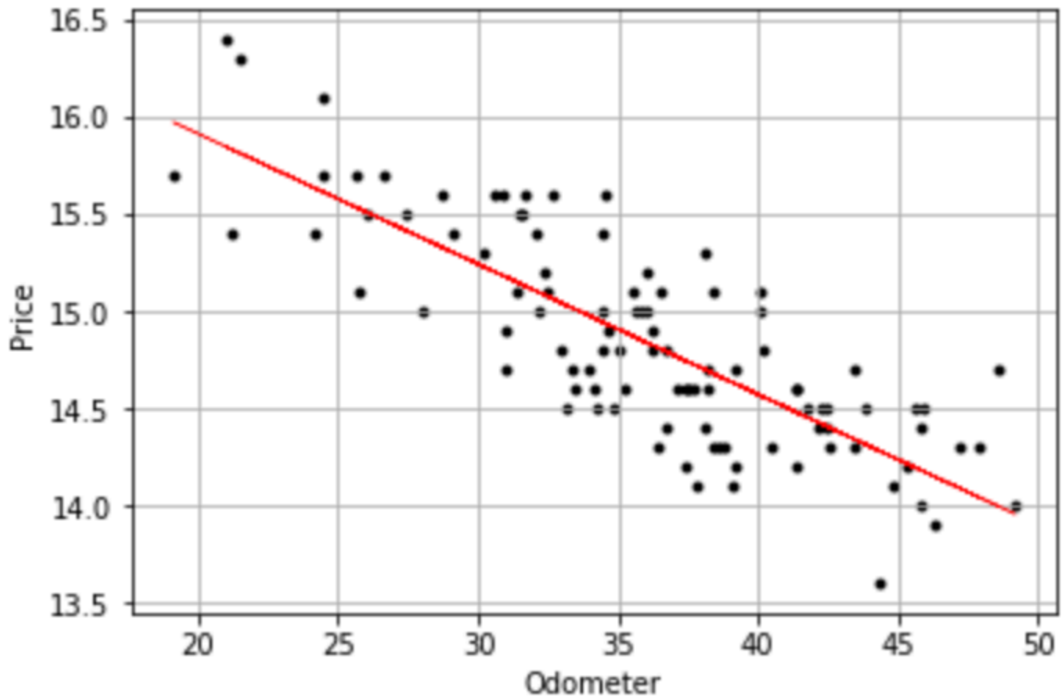


Scatterplot with regression line

```
# predict price
# for all Odometer values in x
```

```
yhat = m1.predict(x_2d)
```

```
plt.figure()
plt.scatter(x,y,c='k',s=9)
plt.plot(x,yhat,color = 'r',
         linewidth = 0.5)
```



Linear Regression with **numpy**

Odometer.csv

	A	B
1	Odometer	Price
2	37.4	14.6
3	44.8	14.1
4	45.8	14
5	30.9	15.6
6	31.7	15.6
7	34	14.7
8	45.9	14.5
9	19.1	15.7
10	40.1	15.1
11	40.2	14.8

X y

Insert a column of ones

	A	B	C
1		Odometer	Price
2	1	37.4	14.6
3	1	44.8	14.1
4	1	45.8	14
5	1	30.9	15.6
6	1	31.7	15.6
7	1	34	14.7
8	1	45.9	14.5
9	1	19.1	15.7
10	1	40.1	15.1
11	1	40.2	14.8

Formula for Linear Regression coefficients

	A	B	C
1		Odometer	Price
2	1	37.4	14.6
3	1	44.8	14.1
4	1	45.8	14
5	1	30.9	15.6
6	1	31.7	15.6
7	1	34	14.7
8	1	45.9	14.5
9	1	19.1	15.7
10	1	40.1	15.1
11	1	40.2	14.8

X y

regression formula

$$b = (X'X)^{-1} X'y$$

$$b = [\, b_0 , b_1 \,]$$

$$= [\, 17.25, -0.0669]$$

Reading csv file with numpy

```python
import numpy as np
import matplotlib.pyplot as plt
```

```python
array1 = np.genfromtxt('Odometer.csv',skip_header=1, delimiter=',')
array1[:5]
```

```
array([[37.4, 14.6],
       [44.8, 14.1],
       [45.8, 14. ],
       [30.9, 15.6],
       [31.7, 15.6]])
```

```python
array1.shape
```

```
(100, 2)
```

	A	B
1	Odometer	Price
2	37.4	14.6
3	44.8	14.1
4	45.8	14
5	30.9	15.6
6	31.7	15.6
	X	y

Reading csv file with numpy

```
import numpy as np
import matplotlib.pyplot as plt
```

```
array1 = np.genfromtxt('Odometer.csv',skip_header=1, delimiter=',')
array1[:5]
```

x y

```
array([[37.4, 14.6],
       [44.8, 14.1],
       [45.8, 14. ],
       [30.9, 15.6],
       [31.7, 15.6]])
```

```
array1.shape
```

```
(100, 2)
```

	A	B
1	Odometer	Price
2	37.4	14.6
3	44.8	14.1
4	45.8	14
5	30.9	15.6
6	31.7	15.6

Reading csv file with numpy

```
import numpy as np
import matplotlib.pyplot as plt
```

```
array1 = np.genfromtxt('Odometer.csv',skip_header=1, delimiter=',')
array1[:5]
```

x y

```
array([[37.4, 14.6],
       [44.8, 14.1],
       [45.8, 14. ],
       [30.9, 15.6],
       [31.7, 15.6]])
```

```
array1.shape
```

```
(100, 2)
```

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

1D arrays

Linear regression with numpy

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

```
# add column of ones to x
```

```
x1 = np.c_[np.ones(100),x]
x1[:5]
```

```
array([[ 1. , 37.4],
       [ 1. , 44.8],
       [ 1. , 45.8],
       [ 1. , 30.9],
       [ 1. , 31.7]])
```

Linear regression with numpy

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

```
# add column of ones to x
```

```
x1 = np.c_[np.ones(100),x]
x1[:5]
```

```
array([[ 1. , 37.4],
       [ 1. , 44.8],
       [ 1. , 45.8],
       [ 1. , 30.9],
       [ 1. , 31.7]])
```

	A	B	C
1		Odometer	Price
2	1	37.4	14.6
3	1	44.8	14.1
4	1	45.8	14
5	1	30.9	15.6
6	1	31.7	15.6

X1 y

Linear regression with numpy

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

```
# add column of ones to x
```

```
x1 = np.c_[np.ones(100),x]
x1[:5]
```

```
array([[ 1. , 37.4],
       [ 1. , 44.8],
       [ 1. , 45.8],
       [ 1. , 30.9],
       [ 1. , 31.7]])
```

	A	B	C
1		Odometer	Price
2	1	37.4	14.6
3	1	44.8	14.1
4	1	45.8	14
5	1	30.9	15.6
6	1	31.7	15.6

X1 **y**

$$b = (X'X)^{-1} X'y$$

Linear regression with numpy

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

```
# add column of ones to x
```

```
x1 = np.c_[np.ones(100),x]
x1[:5]
```

```
array([[ 1. , 37.4],
       [ 1. , 44.8],
       [ 1. , 45.8],
       [ 1. , 30.9],
       [ 1. , 31.7]])
```

$$b = \underbrace{(X'X)^{-1}}_{b1} \underbrace{X'y}_{b2}$$

```
b1 = np.dot(x1.T,x1)
b1 = np.linalg.inv(b1)
b1
```

$(X^TX)^{-1}$

```
array([[ 3.11063002e-01, -8.36030663e-03],
       [-8.36030663e-03,  2.32159802e-04]])
```

```
b2 = np.dot(x1.T,y)
b2
```

X^Ty

```
array([ 1484.1 , 53155.93])
```

Linear regression with numpy

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

```
# add column of ones to x
```

```
x1 = np.c_[np.ones(100),x]
x1[:5]
```

```
array([[ 1. , 37.4],
       [ 1. , 44.8],
       [ 1. , 45.8],
       [ 1. , 30.9],
       [ 1. , 31.7]])
```

$$b = (X'X)^{-1} X'y$$

```
b1 = np.dot(x1.T,x1)
b1 = np.linalg.inv(b1)
b1
```

$(X^TX)^{-1}$

```
array([[ 3.11063002e-01, -8.36030663e-03],
       [-8.36030663e-03,  2.32159802e-04]])
```

```
b2 = np.dot(x1.T,y)
b2
```

X^Ty

```
array([ 1484.1 , 53155.93])
```

```
coeffs = np.dot(b1,b2)
coeffs
```

$(X^TX)^{-1} X^Ty$

```
array([17.24872734, -0.06686089])
```

intercept slope

`

Linear regression with numpy

```
x, y = array1[:,0], array1[:,1]
x[:5]
```

```
array([37.4, 44.8, 45.8, 30.9, 31.7])
```

```
# add column of ones to x
```

```
x1 = np.c_[np.ones(100),x]
x1[:5]
```

```
array([[ 1. , 37.4],
       [ 1. , 44.8],
       [ 1. , 45.8],
       [ 1. , 30.9],
       [ 1. , 31.7]])
```

$$b = (X'X)^{-1} X'y$$

```
b1 = np.dot(x1.T,x1)
b1 = np.linalg.inv(b1)
b1
```

$(X^TX)^{-1}$

```
array([[ 3.11063002e-01, -8.36030663e-03],
       [-8.36030663e-03,  2.32159802e-04]])
```

```
b2 = np.dot(x1.T,y)
b2
```

X^Ty

```
array([ 1484.1 , 53155.93])
```

```
coeffs = np.dot(b1,b2)
coeffs
```

$(X^TX)^{-1} X^Ty$

```
array([17.24872734, -0.06686089])
```

INTERPRET THE SLOPE

Each additional mile decreases the average price by 6.69 cents

Predicting with numpy

Predicting with numpy
requires
coeffs to be a 2D array

```
coeffs.shape
```

```
(2,)
```

1D array

```
coeffs2 = coeffs.reshape(-1,1)
coeffs2
```

```
array([[17.24872734],
       [-0.06686089]])
```

```
coeffs2.shape
```

```
(2, 1)
```

2D array

`

Predict the price of a used car with 40 miles

Prediction = X b

```
coeffs.shape
```

```
(2,)
```

b

```
coeffs2 = coeffs.reshape(-1,1)
coeffs2
```

```
array([[17.24872734],
       [-0.06686089]])
```

```
coeffs2.shape
```

```
(2, 1)
```

X

```
newval = np.array([[1,40]])
newval
```

```
array([[ 1, 40]])
```

```
newval.shape
```

```
(1, 2)
```

X b

```
np.dot(newval,coeffs2)
```

```
array([[14.57429193]])
```

car price is predicted to be 14,574 dollars

Scatterplot with regression line

```
# predict price
# for all Odometer values in x
```

```
yhat = np.dot(x1,coeffs)
```

```
plt.figure()
plt.scatter(x,y,c='k',s=9)
plt.plot(x,yhat,color = 'r',
         linewidth = 0.5)
```