Class Imbalance

Overview

- Imbalance in Classification problems
- Confusion matrix Positive Accuracy
- New Metrics (for class imbalance problems)
- ROC Curve
- Area-under-the-Curve (AUC)
- Example Insurance Data

Class Imbalance

Classification Problems 2 categories labeled as positive and negative (binary classification)

3+ categories (multiclass classification)

Class Imbalance – 2 categories

- Response with 2 categories
 - Positive
 - Negative
- It is more important to accurately predict the Positive category

Square matrix showing

- n. true observations per category (rows)
- n. predicted obs per category (cols)

Class Imbalance

A balanced dataset

Truo	Negative	54
True	Positive	46

Class Imbalance

Let us build a classification model to predict these categories

Truo	Negative	54
True	Positive	46

		Predicted		
		Negative	Positive	
Tuu	Negative			
True	Positive			

We use the model to predict the categories and fill the confusion matrix

		Predicted		
		Negative	Positive	
True	Negative			
	Positive			

Main diagonal will show accurate predictions

		Predicted		
		Negative	Positive	
	Negative			
True	Positive			

Off-main diagonal values will show incorrect predictions (misclassifications)

		Predicted		
		Negative	Positive	
Tuus	Negative			
True	Positive			

Note that we show True categories in the rows and predictions in the columns

		Predicted		
		Negative	Positive	
True	Negative	50	4	
	Positive	6	40	

Model correctly predicted 90 of 100 obs

		Pred	_	
		Negative	Positive	
True	Negative	50	4	54
	Positives	6	40	46

Model correctly predicted 40 of 46 positives (87% accurate rate for positive cases)

- Cancer dataset with 120 patients
- Patients may have cancer (H) or not (DNH)
- Category of interest is H (Positive)
- Dataset has 15 patients with cancer

Class Imbalance

This is an imbalanced dataset

Truo	Negative	105	87.5%
True	Positive	15	12.5%

- Priority is to identify new patients with cancer to treat them as early as possible
- A classification model is built and the results are as follows

		Predicted		
		Negative	with cancer	
True	Negative	102	3	
	with cancer	9	6	

Accuracy rate is (102+6)/120 = 0.90

		Pred		
		Negative	with cancer	
True	Negative	102	3	105
	with cancer	9	6	15

But priority is to identify patients with cancer

		Pred		
		Negative	with cancer	
True	Negative	102	3	105
	with cancer	9	6	15

Model is able to identify 6 out of 15 cancer patients. Positive accuracy is 6/15 = 0.40

		Predicted		
		Negative Positive		
T	Negative	102	3	
True	Positive	9	6	

Total accuracy rate is 108/120 = 0.90 Positive accuracy rate is 6/15 = 0.40

- Priority is to identify new patients with cancer to treat them as early as possible
- A classification model is built and the results are as follows
- A Naive approach is to predict patients as Negative, always

Class Imbalance

An imbalanced dataset

Truo	Negative	105	87.5%
True	Positive	15	12.5%

120

		Pred		
		Negative	with cancer	
True	Negative	105	0	105
	with cancer	15	0	15

Accuracy rate is 105/120 = 0.875Positive accuracy is 0/15 = 0

Counts – Positives, Negatives

Class Imbalance – New Metrics

We need new metrics for the predictions on classification problems with class imbalance

- True Positive, True Negative
- False Positive, False Negative
- False Positive rate (FPR), True Positive rate (TPR)
- ROC, AUC

Class Imbalance – Cancer example

		Predicted		
		Negative	Positive	
True	Negative	102	3	
	Positive	9	6	

Counts

		Predicted		
		Negative	Positive	
T	Negative	True Negatives	False Positives	
True	Positive	False Negatives	True Positives	

Counts

		Predicted			
		Ne	gative	Ро	sitive
Truo	Negative	True	Negatives	False	Positives
True	Positive	False	Negatives	True	Positives

A False is an incorrect prediction

Class Imbalance – New Metrics

		Predicted		
		Negative	Positive prediction	
Truo	Negative		False Positives	
True	Positive	-	True Positives	

- A False is an incorrect prediction
- A False Positive is an incorrect Positive prediction

Class Imbalance – New Metrics

		Predicted		
		Negative	Positive prediction	
Ти	Negative	True Negatives	False Positives	
True	Positive	False Negatives	True Positives	

- A False is an incorrect prediction
- A False Positive is an incorrect Positive prediction
- A False Negative is an incorrect Negative prediction

Take a Covid test

- You get a negative result when in fact you have Covid (False Negative)
- You get a positive result when in fact you do not have Covid (False Positive)

Rates - TPR and Precision

		Predicted		
		Negative	Positive	
True	Negative	True Negatives	False Positives	
	Positive	False Negatives	True Positives	

Counts

		Predicted		
		Negative Positive		
True	Negative	TN	FP	
	Positive	FN	TP	

Short notation

Precision

		Predicted		
		Negative	Positive Predictions	
Truo	Negative	N	FP	
True	Positive	FN	TP	

$$PRECISION = \frac{TP}{Positive}$$
Predictions

How accurate are the positive predictions?

Precision

		Predicted		
		Negative	Positive Predictions	
Truo	Negative	TN	FP	
True	Positive	FN	TP	

$$PRECISION = \frac{TP}{TP + FP}$$

How accurate are the positive predictions?

TP

Positives

True Positive Rate (TPR)

		Pred	_		
		Negative	Positive		
Truo	Negative	TN	FP		
True Cases	Positives	FN	TP	TPR	=

How accurate is model when predicting positive cases?

True Positive Rate (TPR)

		Predicted		
		Negative	Positive	
Truo	Negative	TN	FP	
True Cases	Positives	FN	TP	T_{\cdot}

$$TPR = \frac{TP}{TP + FN}$$

How accurate is model when predicting positive cases?

Class Imbalance

Sometimes we want

Precision to be as large as possible

Other times we want

True Positive Rate to be as large as possible

It depends on the problem

- Dataset with some patients with a tumor
- A classification model is used to predict if a new patient has cancer
- Priority is to identify patients with cancer to treat them as early as possible
- So, let the cancer patients be the positives

.

PREDICTION

TRUE

	Negatives	Positives
Negatives		
Positives	FN	TP

True Positive Rate

$$TPR = \frac{TP}{TP + FN}$$

TRUE

BENIGN	MALIGN	
BENIGN	Cancer patients	TP
MALIGN	Description	TP
Wrong prediction		

Want to keep the False Negatives as small as possible

Want the True Positive Rate as high as possible

Class Imbalance – New Drug Example

- A pharmaceutical company wants to know if its new drug is able to cure a disease
- A classification model is used to predict if the drug is effective or not (binary classification)
- Priority is to identify cases when the drug is effective
- Let 'drug is effective' be the positive case

Class Imbalance – New Drug Example

PREDICTION

		DRUG IS NOT EFFECTIVE	DRUG IS EFFECTIVE	
TRUE	DRUG IS NOT EFFECTIVE		Predict that drug is effective when in fact it is not	
TIVOL	DRUG IS EFFECTIVE		TP	Wrong prediction

Want to keep the False Positives as small as possible

Class Imbalance – New Drug Example

PREDICTION

		DRUG IS NOT EFFECTIVE	DRUG IS EFFECTIVE	
TRUE	DRUG IS NOT EFFECTIVE		Predict that drug is effective when in fact it is not	
TROL	DRUG IS EFFECTIVE		TP	Keep it low
	PRECISION		$=\frac{TP}{TP+FP}$	

Want the Precision as high as possible

More Rates

Confusion Matrix - Rates

		Predicted		
		Negative	Positive	
Truo	Negative	TNR	FPR	
True	Positive	FNR	TPR	

Rates

True Negative Rate

		Predicted		-		
		Negative	Positive			
Truo	Negatives	TN	FP	TNR	=	TN Negatives
True	Positives					

TNR is rate that negatives are accurately predicted

True Positive Rate

		Predicted		
		Negative	Positive	
Truo	Negatives			
True	Positives	FN	TP	

$$TPR = \frac{TP}{\text{Positives}}$$

TPR is rate that positives are accurately predicted

Class Imbalance

		Predicted		_	
		Negative	Positive		
Truo	Negatives	TN	FP	TNR	=
True	Positives	FN	TP	TPR	= 1

$$TNR = \frac{TN}{TN + FP}$$

$$TPR = \frac{TP}{TP + FN}$$

TNR is rate that negatives are accurately predicted TPR is rate that positives are accurately predicted

Class Imbalance

		Pred	-		
		Negative	Positive		
Truo	Negative	TN	FP	TNR	=
True	Positive	FN	TP	TPR	=

$$TNR = 1 - FPR$$

$$TPR = \frac{TP}{TP + FN}$$

TNR = Specificity

TPR = Sensitivity

False Positive Rate

		Predicted		
		Negative	Positive Predictions	
Truo	Negatives	TN	FP	
True	Positive	FN	TP	

$$FPR = \frac{FP}{\text{Negatives}}$$

FPR is the fraction of negatives incorrectly predicted

False Negative Rate

		Predicted		
		Negative Predictions	Positive	
Truc	Negative	TN	FP	
True	Positive	FN	TP	

$$FNR = \frac{FN}{\text{Positives}}$$

FNR is the fraction of positives predicted incorrectly

Class Imbalance – Row complements

FPR + TNR = 1

% negatives
incorrectly predicted
Specificity

TPR + FNR = 1

% positives % positives correctly predicted incorrectly predicted Sensitivity or Recall

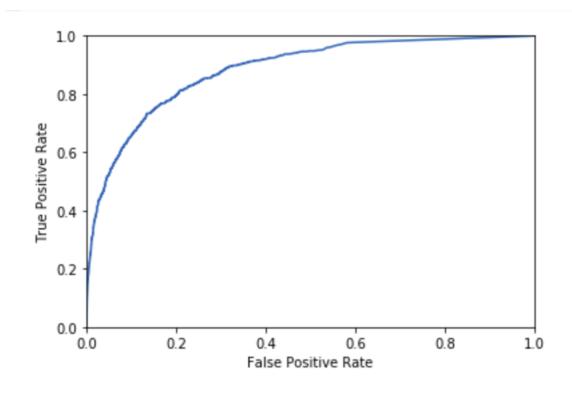
FPR

% negatives incorrectly predicted

TPR

% positives correctly predicted

ROC is a relation between Positive rates

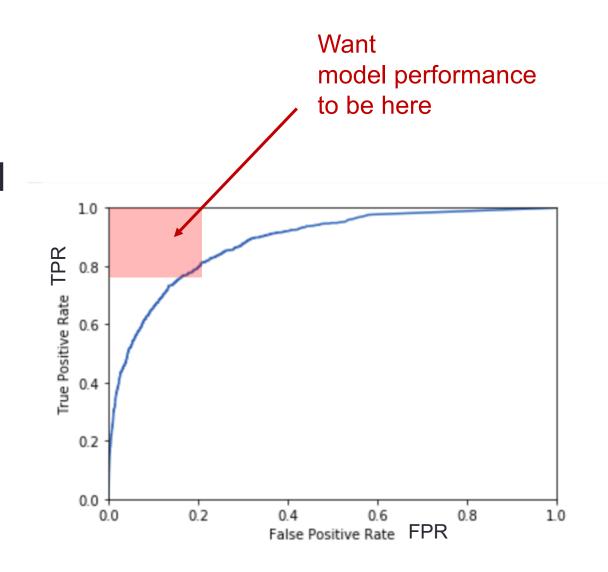


FPR

% negatives incorrectly predicted

TPR

% positives correctly predicted



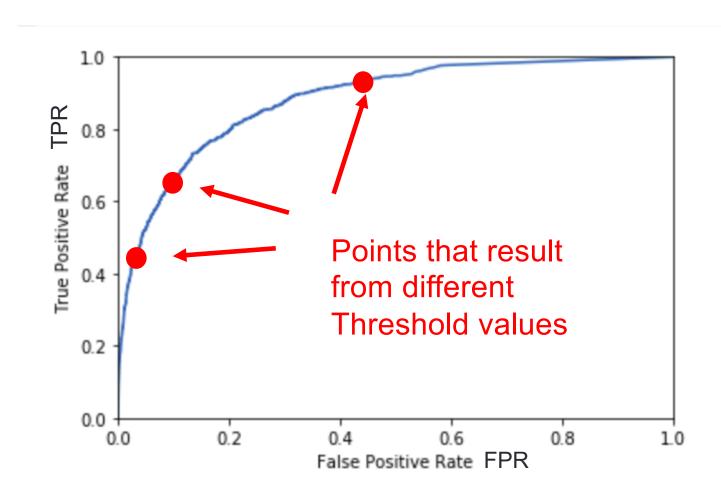
		Predicted	
		Negative	Positive
True	Negative	102	3
	Positive	9	6

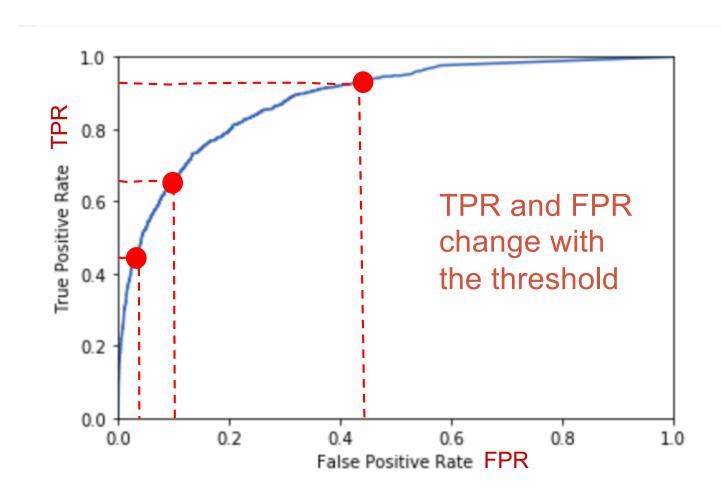
$$FPR = \frac{3}{105} = 0.0286$$

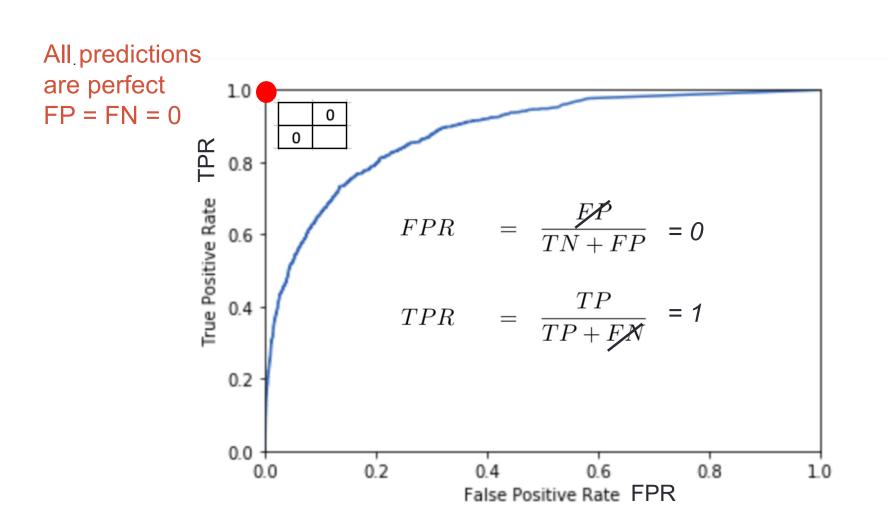
$$TPR = \frac{6}{15} = 0.40$$

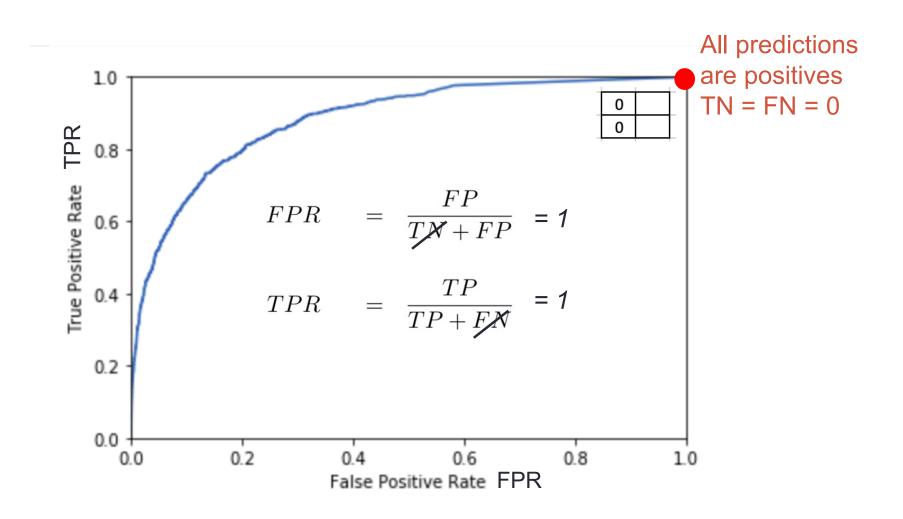
		Predicted	
		Negative	Positive
True	Negative	TN	FP
	Positive	FN	TP

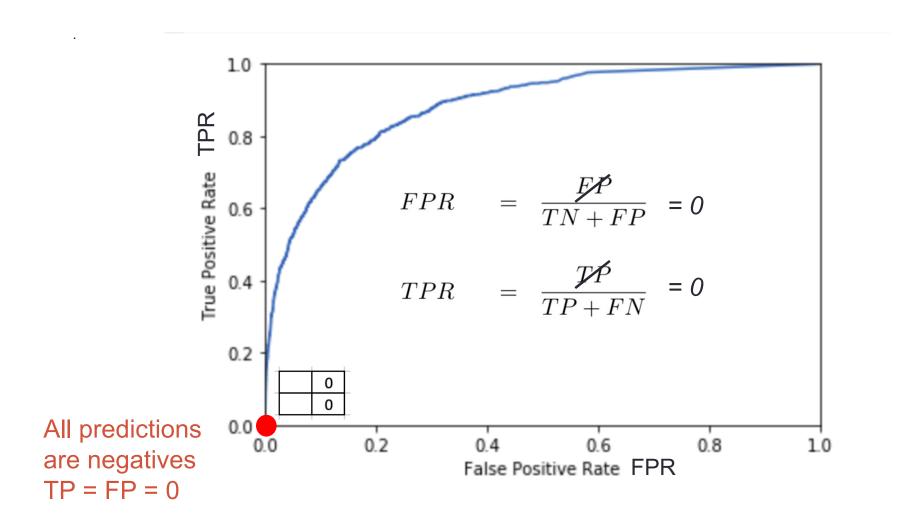
TN, FN, FP, TP values change with the threshold used in the classification model

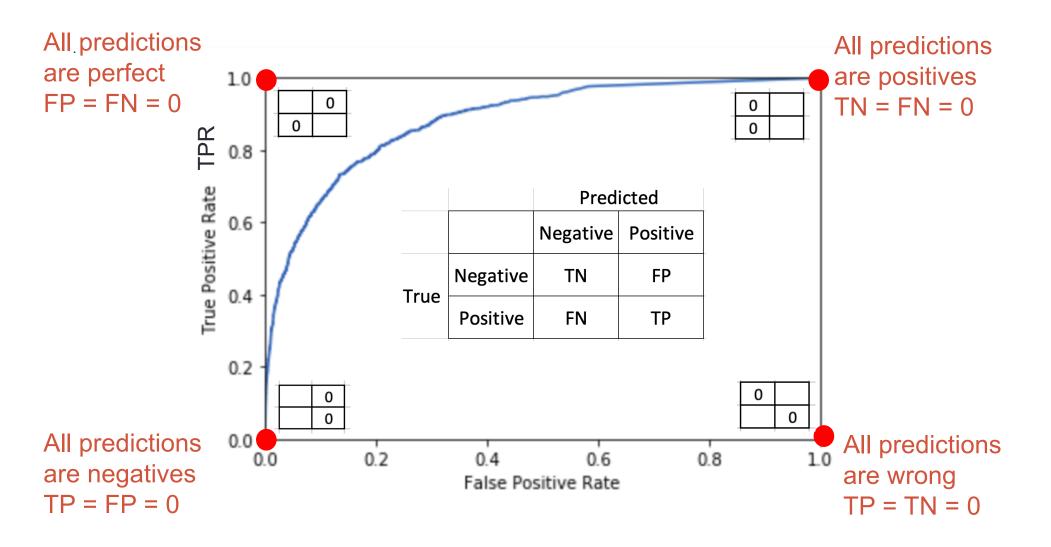


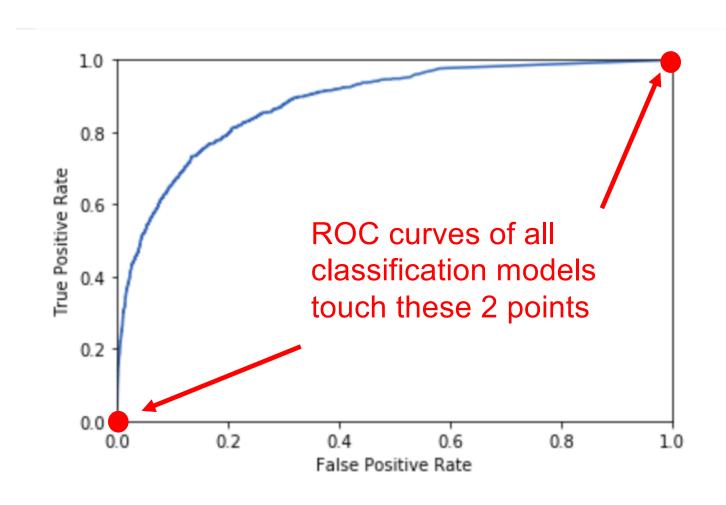


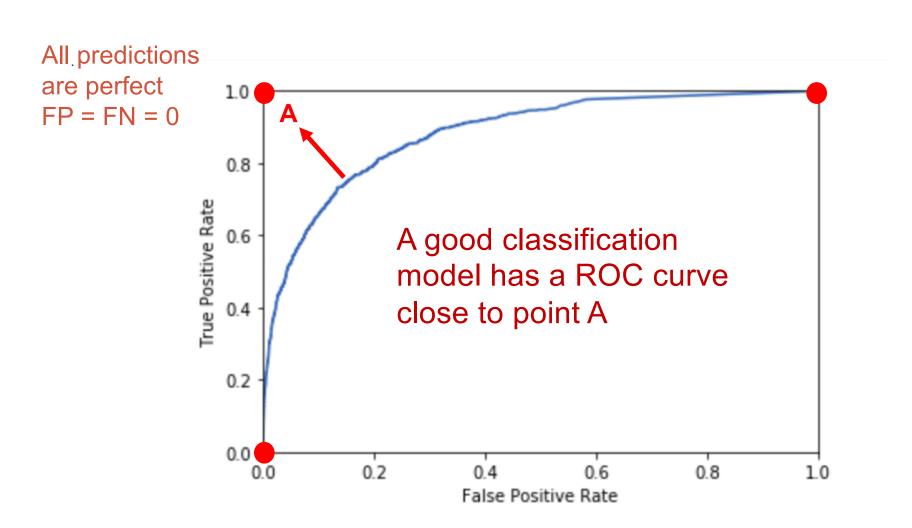


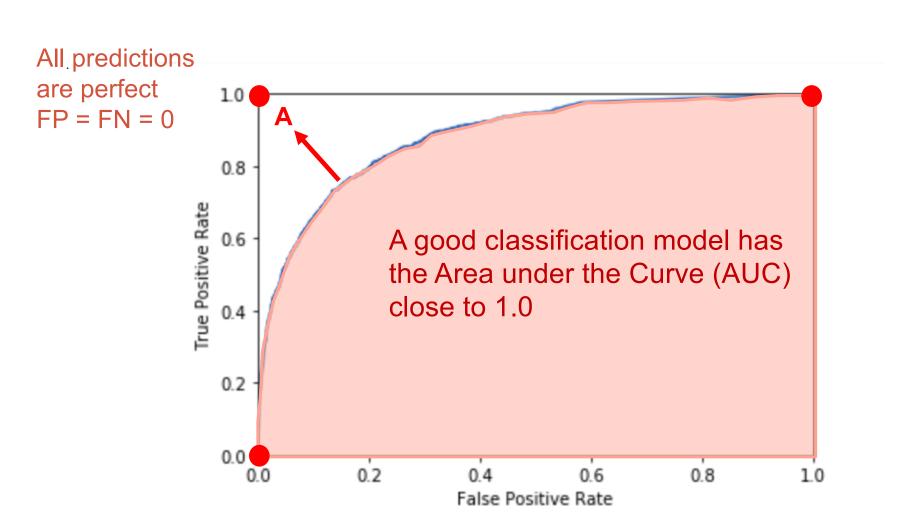


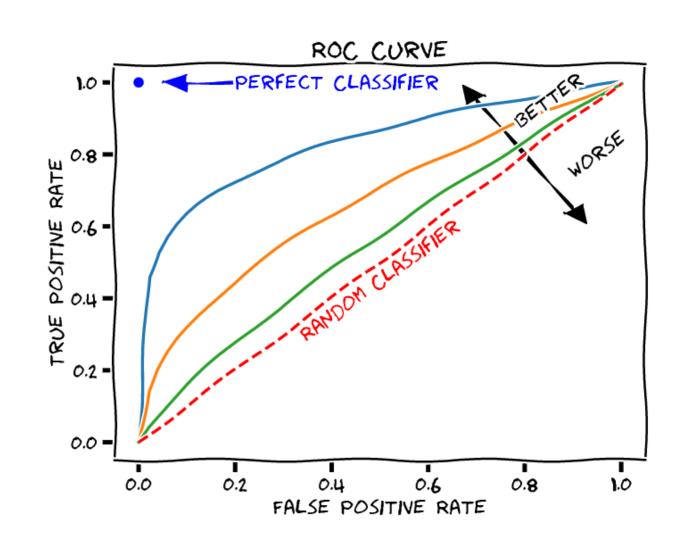




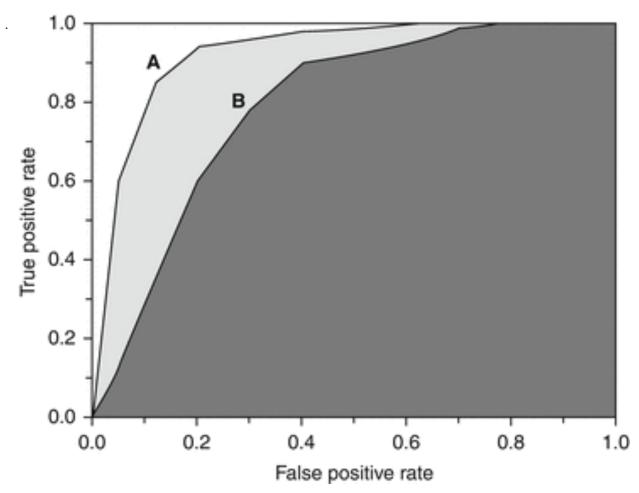








Area under the Curve (AUC)



- A and B are ROC curves from two classification models
- Model A is better classifier than Model B
- The AUC of model A is larger than the AUC of model B

Example – Insurance data

Example – Caravan Insurance Data

- Dataset with n = 5822 customer records
- Each customer has 85 features
 - socio-demographic (variables 1-43)
 - product ownership (variables 44-86)
- Variable 86 (Purchase) is the response.
- It indicates whether the customer purchased the Caravan insurance policy
- About 6% of customers purchased the insurance (94% did not)

Example – Caravan Insurance Data

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model selection import train test split
from sklearn.neighbors import KNeighborsClassifier
from sklearn.linear model import LogisticRegression
from sklearn.metrics import roc curve
from sklearn.metrics import roc auc score
from sklearn.metrics import confusion matrix
from sklearn.metrics import classification report
df = pd.read csv('Caravan.csv')
```

Example – Caravan column names

```
df.columns
Index(['MOSTYPE', 'MAANTHUI', 'MGEMOMV', 'MGEMLEEF', 'MOSHOOFD', 'MGODRK',
       'MGODPR', 'MGODOV', 'MGODGE', 'MRELGE', 'MRELSA', 'MRELOV', 'MFALLEE
Ν',
       'MFGEKIND', 'MFWEKIND', 'MOPLHOOG', 'MOPLMIDD', 'MOPLLAAG', 'MBERHOO
G',
       'MBERZELF', 'MBERBOER', 'MBERMIDD', 'MBERARBG', 'MBERARBO', 'MSKA',
       'MSKB1', 'MSKB2', 'MSKC', 'MSKD', 'MHHUUR', 'MHKOOP', 'MAUT1', 'MAUT
2',
       'MAUTO', 'MZFONDS', 'MZPART', 'MINKM30', 'MINK3045', 'MINK4575',
       'MINK7512', 'MINK123M', 'MINKGEM', 'MKOOPKLA', 'PWAPART', 'PWABEDR',
       'PWALAND', 'PPERSAUT', 'PBESAUT', 'PMOTSCO', 'PVRAAUT', 'PAANHANG',
       'PTRACTOR', 'PWERKT', 'PBROM', 'PLEVEN', 'PPERSONG', 'PGEZONG',
       'PWAOREG', 'PBRAND', 'PZEILPL', 'PPLEZIER', 'PFIETS', 'PINBOED',
       'PBYSTAND', 'AWAPART', 'AWABEDR', 'AWALAND', 'APERSAUT', 'ABESAUT',
       'AMOTSCO', 'AVRAAUT', 'AAANHANG', 'ATRACTOR', 'AWERKT', 'ABROM',
       'ALEVEN', 'APERSONG', 'AGEZONG', 'AWAOREG', 'ABRAND', 'AZEILPL',
       'APLEZIER', 'AFIETS', 'AINBOED', 'ABYSTAND', 'Purchase'],
      dtype='object')
```

(5822, 86)

df.shape

Example – Caravan Insurance Data

```
X = df.drop('Purchase',axis=1)
y = df['Purchase']

y.value_counts()

No     5474
Yes     348
```

```
y.replace(('No', 'Yes'), (0,1), inplace=True)
y.value_counts()

0     5474
1     348
Name: Purchase, dtype: int64

y.value_counts()/569

0     9.620387
1     0.611599     ← Only 6% purchased the insurance
```

Example – KNN model

Example – KNN

```
yhat = np.zeros(m)
yhat[probabs >= 0.5] = 1
                                                  ← 0.50 is the threshold (default)
conf_matrix1 = pd.crosstab(y_test,yhat,
                              rownames=['y_test'],
                              colnames = ['predictions'])
conf_matrix1
predictions
            0.0
               1.0
    y_test
        0 1643
                0
                0
           104
                         tpr1 = 0/(104+0)
                         tpr1
                                                      ← Accuracy Rate for positive category
                         0.0
                         fpr1 = 0/(1643+0)
                         fpr1
                         0.0
```

Example – KNN

```
yhat = np.zeros(m)
yhat[probabs >= 0.05] = 1
                                                   ← 0.050 threshold (arbitrarily selected)
yhat = yhat.astype(int)
conf_matrix2 = pd.crosstab(y_test,yhat,
                              rownames=['y_test'],
                             colnames = ['predictions'])
conf_matrix2
predictions
            0
               1
    y_test
                                 changing the threshold
        o 870 773
                                 results in new
           36 68
                                 confusion matrix and
                                 FN, TP, FP, TN values
```

Example – KNN

predictions 0 1

y_test

0 870 773

1 36 68

model predicts that 773 customers would buy the insurance (predictions = 1) but they actually did not (y test = 0)

```
tpr2 = 68/(68+36)
tpr2
```

0.6538461538461539

← New accuracy rate for positive category

```
fpr2 = 773/(773+870)
fpr2
```

0.4704808277541083

Example – ROC Curve for KNN model

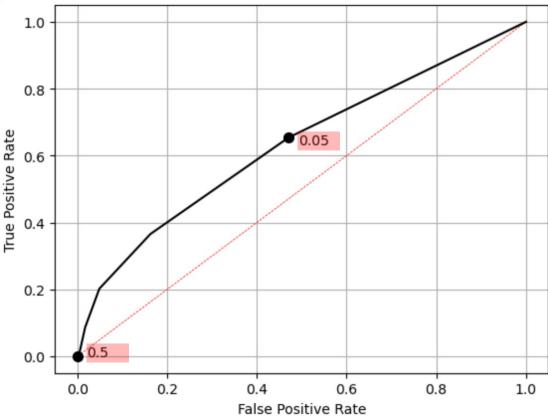
```
from sklearn.metrics import roc_curve

fpr,tpr,thresholds = roc_curve(y_test,probabs)
```

```
# plot 45-degree line
xaxis = yaxis = [0,1]
plt.plot(xaxis,yaxis,'r--',linewidth=0.5)

# highlight two thresholds
plt.plot(0.47,0.6538, 'ko',ms=7)
plt.annotate(0.05,(1.05*0.47, 0.97*0.6538))
plt.plot(0,0, 'ko',ms=7)
plt.annotate(0.50,(0.02,0))

# display ROC curve
plt.plot(fpr,tpr,'k')
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
```



Example – ROC Curve and AUC

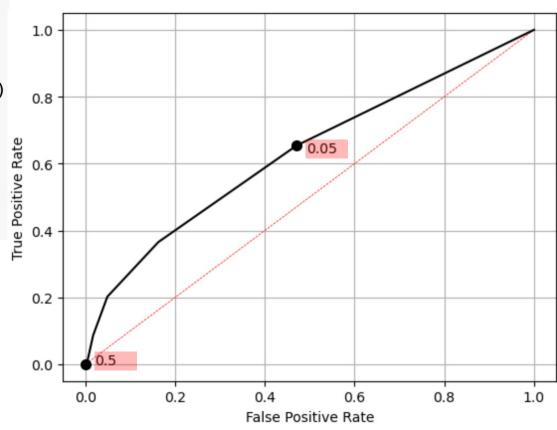
```
from sklearn.metrics import roc_curve
fpr,tpr,thresholds = roc curve(y test,probabs)
```

```
# plot 45-degree line
xaxis = yaxis = [0,1]
plt.plot(xaxis,yaxis,'r--',linewidth=0.5)
# highlight two thresholds
plt.plot(0.47,0.6538, 'ko',ms=7)
plt.annotate(0.05,(1.05*0.47, 0.97*0.6538))
plt.plot(0,0, 'ko',ms=7)
plt.annotate(0.50,(0.02,0))
# display ROC curve
plt.plot(fpr,tpr,'k')
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
```

from sklearn.metrics import roc_auc_score

roc_auc_score(y_test,probabs)

0.6323007865536776

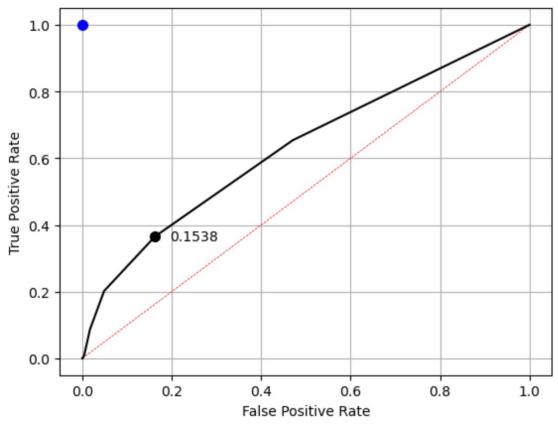


Example – Find best threshold

```
idx = np.argmax(tpr-fpr)

print('idx =',idx)
print('fpr =',fpr[idx],', tpr =',tpr[idx])
print('best threshold =',thresholds[idx])

idx = 5
fpr = 0.16250760803408398 , tpr = 0.3653840
best threshold = 0.15384615385
```

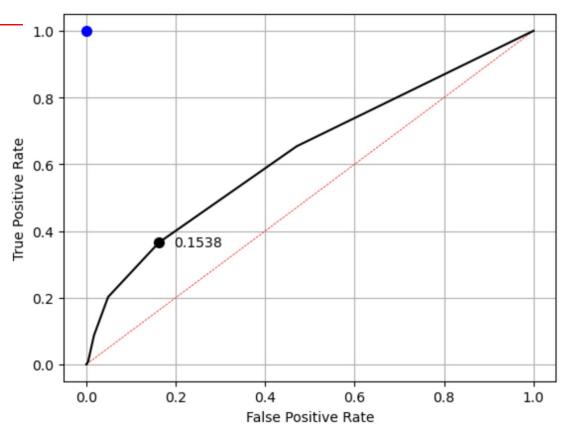


Example – Find best threshold

```
idx = np.argmax(tpr-fpr)

print('idx =',idx)
print('fpr =',fpr[idx],', tpr =',tpr[idx])
print('best threshold =',thresholds[idx])

idx = 5
fpr = 0.16250760803408398 , tpr = 0.3653844
best threshold = 0.15384615384615385
```



Example – Logistic Regression

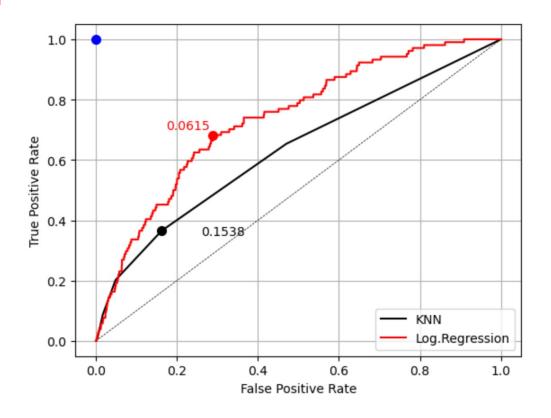
```
logistic regression = LogisticRegression(max iter=10000)
logistic_regression.fit(X_train, y_train)
logistic_regression.score(X_test,y_test)
0.9381797366914711
probabs2 = logistic regression.predict proba(X test)
probabs2
array([[0.98889456, 0.01110544],
       [0.94287391, 0.05712609],
       [0.97454586, 0.02545414],
       [0.98280774, 0.01719226],
       [0.90660734, 0.09339266],
       [0.92555056, 0.07444944]])
probabs2 = logistic_regression.predict_proba(X_test)[:,1]
fpr2,tpr2,thresholds2 = roc curve(y test,probabs2)
```

Example – Logistic Regression best threshold

```
idx2 = np.argmax(tpr2-fpr2)

print('idx =',idx2)
print('fpr =',fpr2[idx2],', tpr =',tpr2[idx2])
print('best threshold =',thresholds2[idx2])

idx = 151
fpr = 0.28910529519172246 , tpr = 0.6826923076
best threshold = 0.06149132365559058
```



Example – Logistic Regression

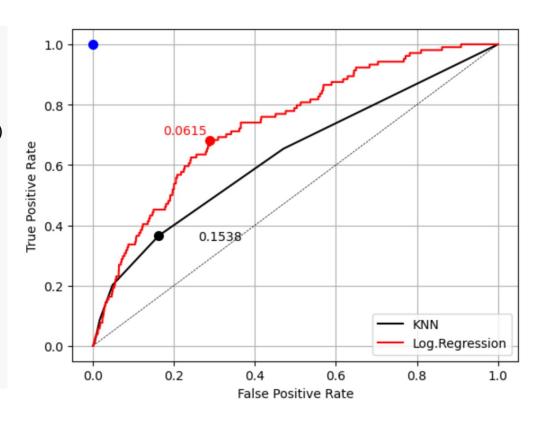
```
idx2 = np.argmax(tpr2-fpr2)

print('idx =',idx2)
print('fpr =',fpr2[idx2],', tpr =',tpr2[idx2])
print('best threshold =',thresholds2[idx2])

idx = 151
fpr = 0.28910529519172246 , tpr = 0.6826923076
best threshold = 0.06149132365559058
```

Logistic Regression ROC Curve above KNN ROC Curve

```
plt.plot(xaxis, yaxis, 'k--', linewidth=0.5)
# KNN
plt.plot(0,1, 'bo',ms=7)
plt.plot(fpr[idx], tpr[idx], 'ko',ms=7)
plt.annotate(value,(1.6*fpr[idx], 0.96*tpr[idx]))
plt.plot(fpr,tpr,'k',label = 'KNN')
# Logistic Regression
value2 = thresholds2[idx2].round(4)
plt.plot(fpr2[idx2], tpr2[idx2], 'ro',ms=7)
plt.annotate(value2,
             (0.6*fpr2[idx2], 1.03*tpr2[idx2]),
             color = 'r')
plt.plot(fpr2,tpr2,'r',label = 'Log.Regression')
plt.xlabel('False Positive Rate')
plt.vlabel('True Positive Rate')
plt.legend(loc=4)
```



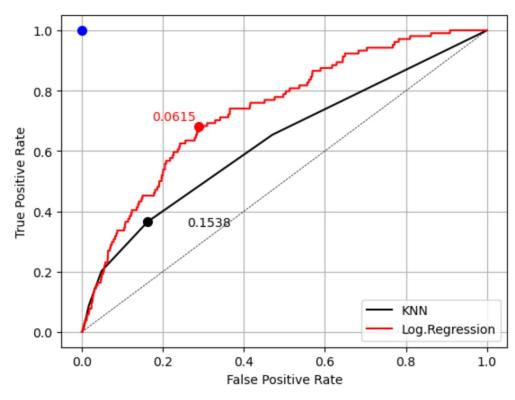
Example – Logistic Regression

```
idx2 = np.argmax(tpr2-fpr2)

print('idx =',idx2)
print('fpr =',fpr2[idx2],', tpr =',tpr2[idx2])
print('best threshold =',thresholds2[idx2])

idx = 151
fpr = 0.28910529519172246 , tpr = 0.6826923076
best threshold = 0.06149132365559058
```

Logistic Regression ROC Curve above KNN ROC Curve



Example – AUC Comparison

```
from sklearn.metrics import roc_auc_score
# AUC for KNN
auc1 = roc_auc_score(y_test,probabs)
auc1
0.6323007865536776
# AUC for Logistic Regression
auc2 = roc_auc_score(y_test,probabs2)
auc2
0.7385586872044572
```

Example – ROC Curve

- The ROC curve is a tool to find the best threshold (cutoff) value
- We want a threshold where TPR is large while the FPR is small
- To find the best threshold value find all fpr,tpr for a large number of thresholds using fpr, tpr, thresholds = roc_curve(y_test, probabs)
- The best point on the ROC Curve is found at row idx using idx = np.argmax(tpr - fpr)